Smooth orthogonal and Non-orthogonal Factorizations in Dynamical Systems

It is well-known that constant matrix factorizations (LU, QR, SVD, Schur) are central tools in matrix computations, linear algebra and related areas. They are very useful in applications including: computation of orthonormal bases of invariant subspaces, low rank approximations, compression and information retrieval.

A natural extension is the study of such factorizations of matrix functions $A(s)$, where $s$ is a parameter in some interval, and the corresponding factors are at least as smooth as the function $A$ itself. These factorizations are crucial in several areas of dynamical systems; in particular, they allow for the computation and smooth continuation of center-stable or center-unstable subspaces of equilibrium points or periodic orbits [1,3,4]. Similarly, they can be used to solve general linear and nonlinear eigenvalue problems [5]. Practical applications of smooth factorizations include space mission designs [6], population dynamics [7] and vibration of fluid-solid structures [9].

For reasons of computational stability, factorizations are typically performed so that the factors are orthogonal matrices for every value of the parameter under consideration, as in Schur factorizations [2,8]. These factorizations although very stable, may be in general very expensive to compute. The question is whether sacrificing orthogonality for more general matrices can still give some stability and yet reduce the computational cost. In this direction, smooth LU factorizations of matrix functions with an eye to dynamical systems seem to be an excellent option and an interesting topic of research. We know that for constant matrices, LU is feasible and more easily realizable than other factorizations. The strategy would be to use Gauss-Newton type methods and pseudoarclength continuation to smoothly extend the factorization as the parameter $s$ varies. Some questions to be answered for these non-orthogonal factorizations are: under what conditions on $A(s)$ does the factorization exist? Is the factorization unique? How can we guarantee the smoothness of the factors? How does the computational cost compare to that in orthogonal factorizations?

To apply these factorizations in dynamical systems, the student will have to deal with more advanced concepts like monodromy matrices, Floquet theory, etc. This would be truly challenging problems for the most ambitious and talented students, and for the continuation of REU projects in the next years.

Prerequisites: A basic background on linear algebra and differential equations. Optimally, some experience with dynamical systems is desirable, but not required.

References


