**Properties of Graphs Associated to Groups (L. Reid)**

There are many graph-theoretical constructs that can be associated to a group. Investigating which properties these graphs possess and how these properties relate to the algebraic properties of the group provides a broad range of opportunities for undergraduate research. This project is appropriate for undergraduates, has a high probability of success, and offers a wide range of possible levels of completion. This project involves the use of technology (primarily *Mathematica* and/or *GAP*), both to be used as a computational tool and to generate data to assist in the formulation of conjectures. Students in our previous REU have successfully worked on similar problems, and in some cases their results have been submitted to or published in refereed journals.

One of the most natural constructions is the subgroup lattice of a group. The vertices are subgroups of the group and there is an edge between two vertices if and only if one subgroup is contained in the other with no intermediate subgroup properly between them. The graph-theoretical properties of these graphs have been extensively investigated in prior REU's and master's theses [4,5,7,13,15], but there are still some topics that could be investigated (e.g. graph pebbling or cop number).

The subgroup graph is also a lattice. Determining the Möbius function of that lattice is critical in determining the probability that $k$ distinct elements generate a group. In her thesis [6], Christie Tosh Bowerman has addressed this problem for a number of groups, but there are many families of groups left to investigate.

A second graph associated to a group $G$ is the non-commuting graph. This is the graph whose vertices are the elements of $G - Z(G)$ (namely the elements that do not commute with everything) with an edge between the vertex associated to $x$ and that associated to $y$ if and only if $xy \neq yx$. Some properties of this graph have been investigated for certain groups [1,2,8], interesting conjectures have been made [17], and these graphs have been the subject of a prior REU [9,10,13,15,16], but many topics remain to be studied.

A third graph is the subgroup intersection graph, whose vertices are the non-trivial proper subgroups and two vertices are connected by an edge if and only if they intersect non-trivially. Ahmadi and Taeri [3] and independently Kayacan and E. Yaraneri [11] characterized those groups whose subgroup intersection graphs are planar. Last summer's REU group is well on its way to determining those groups whose subgroup intersection graphs are of genus one [12].

There are many other candidate graphs to investigate including the non-cyclic graph, variants on the noncommuting graph, the conjugacy graph, and G-graph.

In the author's experience, the synergism between the group-theoretical and graph-theoretical aspects of these problems makes this project particularly appealing to undergraduates. Upon completion of the project, participants should have a greatly deepened understanding of both group theory and graph theory.

*Prerequisites:* Calculus and one semester of abstract algebra (including group theory). Some exposure to graph theory is desirable, but not required.

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