Higher Codimension Bifurcations in Models of Neuronal Systems (J. Rebaza)

A common feature in models of neuronal activity is the transition from spiking to bursting. This is often related to a complex bifurcation structure of the corresponding mathematical model; in particular, codimension-1 bifurcations are not sufficient to fully describe the dynamics in certain regions of the phase portrait and one must study higher codimension bifurcations and their effect on the stability of equilibrium points and limit cycles. Following related work by several authors [e.g. 1, 2, 4, 5], we plan to model neuronal activity via a set of differential equations (with fast and slow time scales), provide with a full stability analysis, and study existence of bifurcations of codimension one and higher, including: Hopf, saddle-node, Bogdanov-Takens, and cusp bifurcations. While previous research is mostly focused on the graphical analysis of such bifurcations, we aim at providing rigorous proofs of existence of such bifurcations, as well as some detailed numerical simulations using Xppaut [3] and Matlab.

At a second level of research, we plan to study the existence and dynamics of so called torus canards [2]. Models in this case have a torus bifurcation (as opposed to Hopf bifurcation) in the full system and a saddle-node bifurcation of periodic orbits in the fast system. Periodic orbits typically show a rapid transition from small to large amplitude, in what is known as a canard explosion. Among other things, we will study a possible relationship between torus canards and the transitions between states of spiking and bursting.

Prerequisites: A basic background on differential equations and linear algebra, in addition to the calculus sequence. Optimally, some programming experience is desirable, but not required.

References

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[4] H. Gu, B. Pan, G. Chen, L. Duan, *Biological experimental demonstration of bifurcations from bursting to spiking predicted by theoretical models*, Nonl. Dynamics (2014) DOI 10.1007/s/11071-014-1447-5.

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