Finite Point Configurations (S. Senger)

There are many classical problems in discrete geometric combinatorics which ask about the distribution of point configurations in subsets of various vector spaces, as familiar as the plane, or as exotic as a vector space over a large finite field. One of the most famous, due to Erdős in 1946, asks for bounds on how often any fixed distance can occur in a large point set in the plane. This question and its relatives has seen much activity in the last few decades [1, 2, 3, 4].

The author proposes additional investigation for functionals other than Euclidean distance, such as dot products, or various non-Euclidean distances, particularly in special classes of point sets which have some built in structure (or lack thereof). These explorations elucidate which arguments in the main questions are essential, and which techniques have met their limit of utility.

These problems are ideal for REUs as their elementary statements require very little background. In fact, the author presents many of the fundamental results in this area to gifted high school students in a summer program each year. In addition to his own regular contribution to this area, the author has also successfully mentored an undergraduate project which obtained novel bounds on how often a pair of dot products can occur in triples of points in a wide class of subsets in the plane, which are currently in preparation for publication.

Prerequisites: Basic geometry and abstract algebra.

References

[1] P. Brass, W. Moser, J. Pach, *Research problems in discrete geometry*, Springer, 18, 2005, 499 pp.

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[3] L. Guth and N. H. Katz, *On the Erdős distinct distance problem in the plane*, Arxiv preprint arXiv:1011:4015, 2010 - arxiv.org.

[4] J. Spencer, E. Szemerédi, and W. T. Trotter, *Unit distances in the Euclidean plane*, B. Bollobás, editor, "Graph Theory and Combinatorics," pages 293-303, Academic Press, New York, NY, 1984.