Math 540/640: Statistical Theory I

HW #4

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Problem 1: Each week a lottery machine selects three balls, each with one digit from 0 to 9. Suppose the digits are i_1, i_2 and i_3. Then let $X((i_1; i_2; i_3)) = 100i_1 + 10i_2 + i_3$. For example, X((0; 1; 2)) = 12 and X((1; 0; 5)) = 105. The value of X is the winning number of the lottery.

a. Identify the distribution of X.

b. Suppose you buy the number 777 (because you think 777 is your lucky number) for 54 consecutive weeks. Let Y be the number of times you win. Identify the distribution of Y . What is P(Y = 0)?

Problem 2: A civil engineer is studying a left-turn lane that is long enough to hold 7 cars. Let X be the number of cars in the lane at the end of randomly selected red light. The engineer believes that the probability that $X = x$ is proportional to $(x + 1)(8 - x)$ for $x = 0, \cdots, 7$ (the possible values of X).

a. find the probability function of X.

b. find the probability that X will be at least 5.

Problem 3: suppose the p.d.f. of a random variable X is $f(x) = 4(1 - x^3)/3$ for $0 < x < 1$, and $f(x) = 0$ otherwise. Sketch this p.d.f. and find

(a). $P(X < 1/2)$. **(b).** $P(1/4 < X < 3/4)$ **(c).** $P(X > 1/3)$.

Problem 4: Suppose the pdf of X is given by $f(x) = ax^3e^{-x}$ for $x > 0$, and $f(x) = 0$ otherwise. Find the value of the constant a.

Problem 5: Suppose $X \sim Unif(-1; 1)$. Let $Y = X^2$ and $Z = X^3$. Find (a). the p.d.f. of Y , and (b). the p.d.f. of Z.