Math 540/640: Statistical Theory I

HW #5

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Problem 1: Suppose (X, Y) has the joint p.d.f.

$$f(x, y) = \begin{cases} x + y & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Let Z = X +Y. Compute the values of CDF and pdf of Z at a when $1 < a < 2$

Problem 2: Suppose (X, Y) has the joint p.d.f.

$$f(x, y) = \begin{cases} e^{-x-y} & \text{if } 0 < x, 0 < y \\ 0 & \text{otherwise} \end{cases}$$

Let Z = X/Y. Compute $P(Z < a)$ and from there find the pdf of Z.

Problem 3: Suppose (X, Y) has the joint p.d.f.

$$f(x, y) = \begin{cases} c(x + y^2) & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute the joint distribution function of (X, Y), i.e., F(x, y), for $0 < x < 1$ and $0 < y < 1$.

Problem 4: Suppose the joint distribution function of (X; Y) is

$$F(x, y) = \frac{e^x}{1 + e^x} \frac{e^{2y}}{1 + e^{2y}}$$

Find $F_X(x)$, $F_Y(y)$ and $f(x, y)$. [hint: use equations on page 39 of lecture notes 4.]

Problem 5: Let's do some marginal density calculation. (a). Assume the joint pdf of (X, Y) is the one given in problem 1. Compute the pdf of X. (b). Assume the joint pdf of (X, Y) is the one given in problem 2. Compute the pdf of Y.