Math 540/640: Statistical Theory I

HW #6

Instructor: Songfeng (Andy) Zheng

Problem 1: Suppose (X, Y) has the joint p.d.f.

$$f(x,y) = \begin{cases} 6xy^2 & \text{if } 0 < x < 1; 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a). Compute the marginal density of X and Y , and then show X and Y independent.

(b). Without doing part (a), use Factorization theorem to show X and Y are independent.

Problem 2: Suppose (X, Y) has the joint p.d.f.

$$f(x,y) = \begin{cases} x + y & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $f_X(x|Y = 0.5)$ and then $P(0.5 < X < 0.75|Y = 0.5)$.

Problem 3: Suppose X is a standard normal random variable. The conditional pdf of Y given $X = x$ is

$$f_Y(y|X = x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}$$

Find the conditional pdf of X given $Y = y$.

Problem 4: Suppose the joint distribution function of (X, Y) is

$$f(x,y) = \frac{1}{2\pi} e^{-\frac{y^2 + x^2}{2}}$$

Let $(U, V) = (2X + Y, X - 3Y)$. Find the joint pdf of (U, V).

Problem 5: Suppose (X, Y) has the joint p.d.f.

$$f(x,y) = \begin{cases} \frac{1}{\pi} & \text{if } x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $R = \sqrt{X^2 + Y^2}$ and $\Theta = \arctan(Y/X)$

(a). Find the joint pdf of (R, Θ). Use factorization theorem to argue that (R, Θ) are independent.

(b) Find out the marginal pdf of R.

(c) Directly use

$$\iint_{x^2+y^2\leq a^2} f(x,y)dxdy$$

to compute $P(R \leq a)$. Then use your result to compute the pdf of R. Does your answer agree with (b)?