Math 540/640: Statistical Theory I (Spring 2011)

HW #7

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Problem 1: Let $X \sim Unif(-1, 2)$.

(a) Use the definition to compute E(X) and Var(X)

(b). Let $Y = (X + 1)/3$. Then argue $Y \sim Unif(0; 1)$. Thus we know E(Y) = 1/2 and Var(Y) = 1/12. Note that X = 3Y - 1. Use E(Y) = 1/2; Var(Y) = 1/12 to compute E(X) and Var(X): Compare the results with (a). What do you see?

Problem 2: Suppose (X, Y) has the joint p.d.f. $f(x, y) = \frac{1}{\pi} I(x^2 + y^2 \leq 1)$. Compute $E[\frac{1}{\sqrt{X^2+Y^2}}]$.

Problem 3: Let $X \sim Unif(0; 1)$ and $Y \sim N(0; 1)$. What is E[2X -Y + 1]? Now if further assume X and Y are independent, what is Var(2X-Y +1)?

Problem 4: Let X be a random variable with the pdf $f(x) = xe^{-x} I(x > 0)$.

(a) Compute E(X) and Var(X).

(b) If $X_1, X_2, \cdots, X_n$ are i.i.d. random variables that have the same pdf as X. Use (a) to find $E[\frac{X_1+X_2+\cdots+X_n}{n}]$ and $Var[\frac{X_1+X_2+\cdots+X_n}{n}]$

Problem 5: Let X and Y be independent N(0; 1) random variables. Let $Z = \sqrt{X^2 + Y^2}$.

(a). Derive the marginal pdf of Z and then using the marginal pdf to compute $E[Z^2]$.

(b) Can you propose a different way other than that in (a) to compute $E[Z^2]$.

(c) Compute $E[Z]$.