Math 540/640: Statistical Theory I

HW #9

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Problem 1: (From $N(0,1)$ to χ^2_k)

(a) Let $X \sim N(0,1)$. Let $Y = X^2$. Compute the MGF of Y and show that Y is a χ^2 random variable with degrees of freedom 1.

(b) Now suppose $X_1, X_2, \cdots, X_k$ are k iid $N(0,1)$ and let $Z = \sum_{i=1}^k X^2$. Use the conclusion of (a) to argue that $Z \sim \chi^2_k$.

Problem 2: Suppose $X \sim N(1,4)$ and $Y \sim N(2,1)$ and $Z \sim N(2,3)$. Assume X, Y and Z are independent. Find the distribution of (a) $X - Y$, (b) $X + Y - 2Z$ and (c) $X - Y - 2Z$.

Problem 3: (From Exp(1) to Chi-squares) Suppose X_i, $i = 1, 2, \cdots, n$ are independent $Exp(1)$. Let $Y = 2(X_1 + X_2 + \cdots + X_n)$. Show that Y is a chi-square random variable. What is its degrees of freedom?

hint: compute the MGF of Y.

Problem 4: Suppose X and Y are independent Poisson random variables. $X \sim Poi(1)$ and $Y \sim Poi(2)$. Identify the conditional probability of X given $X + Y = n$.

hint: use the definition

$$P(X = k | X + Y = n) = \frac{P(X = k, Y = n - k)}{P(X + Y = n)}$$

to compute the conditional probability function of X.