Math 543/653: Stochastic Modeling

HW#2

Instructor: Songfeng (Andy) Zheng

Note: in the HW, rate is the reciprocal of mean.

Problem 1. Let X_i, $i = 1, \cdots, n$ be independent continuous random variables, with X_i having the hazard function $r_i(t)$. Let T be independent of this sequence, and suppose that $\sum_{i=1}^n P(T = i) = 1$. Prove that the hazard function for $X = X_T$ is

$$r(t) = \sum_{i=1}^n r_i(t) P(T = i | X > t).$$

Problem 2. Let X_1 and X_2 be independent exponential random variables with rates μ_1 and μ_2. Define

$$X_{(1)} = \min(X_1, X_2)$$

and

$$X_{(2)} = \max(X_1, X_2).$$

Find

(a) $E[X_{(1)}]$ and $Var[X_{(1)}]$

(b) $E[X_{(2)}]$ and $Var[X_{(2)}]$

Problem 3. Let X be an exponential random variable with rate λ. Let c be a positive constant.

(a) Use the definition of conditional expectation to calculate $E(X|X > c)$.

(b) Use the memoryless property to determine $E(X|X > c)$.

(c) Use the definition of conditional expectation to calculate $E(X|X < c)$.

(d) Prove the following identity

$$E(X) = E(X|X < c)P(X < c) + E(X|X > c)P(X > c).$$

(e) Use the conclusion in (a) or (b) and the identity in (d) to calculate $E(X|X < c)$.

Problem 4. The life time of A's dog and cat are independent exponential random variables with rates λ_d and λ_c. One of them just died, find the expected additional life time of the other pet.

Problem 5. Let X and Y be independent exponential random variables with respective rates λ and μ with $\lambda > \mu$. Let $c > 0$.

(a) Show that the conditional density of X given that $X + Y = c$ is

$$f_{X|X+Y=c} = \frac{(\lambda - \mu)\exp[-(\lambda - \mu)x]}{1 - \exp[-(\lambda - \mu)c]}, \qquad 0 < x < c.$$

(b) Use (a) to find $E[X|X + Y = c]$.

(c) Find $E[Y|X + Y = c]$.