Math 543/653: Stochastic Process

HW #5

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Problem 1. A Markov Chain $X_0, X_1, X_2, \cdots$ state space $\{0, 1, 2\}$, and has the transition probability matrix

$$P = \begin{Vmatrix} 0.1 & 0.1 & 0.8 \\ 0.2 & 0.2 & 0.6 \\ 0.3 & 0.3 & 0.4 \end{Vmatrix}.$$

In the long run, what fraction of time does the process spend in state 1?

Problem 2. Consider a Markov chain with state space $\{0, 1, 2, 3\}$. Suppose the transition probability matrix is

$$P = \begin{Vmatrix} 0.1 & 0.5 & 0 & 0.4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{Vmatrix}.$$

Determine the limiting distribution for the process.

Problem 3. Consider a Markov chain with state space $\{0, 1, 2, 3, 4, 5\}$, and the transition probability matrix is

$$P = \begin{Vmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{Vmatrix}.$$

where $\alpha_i > 0$, and $\sum \alpha_i = 1$. Determine the limiting probability of being in state 0.

Problem 4. For a finite state regular Markov chain, determine the following limits in terms of the transition probabilities $P = \|P_{ij}\|$ and the limiting distribution $\pi = (\pi_0, \cdots, \pi_N)$:

a. $\lim_{n\to\infty} P(X_{n+1} = j | X_0 = i)$.

b. $\lim_{n\to\infty} P(X_n = k, X_{n+1} = j | X_0 = i)$.

c. $\lim_{n\to\infty} P(X_{n-1} = k, X_n = j | X_0 = i)$.

Problem 5. Consider a Markov chain with state space $\{0, 1, \cdots, N\}$, and the transition probability matrix is

$$P = \begin{Vmatrix} p_0 & p_1 & p_2 & \cdots & p_N \\ p_N & p_0 & p_1 & \cdots & p_{N-1} \\ p_{N-1} & p_N & p_0 & \cdots & p_{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_1 & p_2 & p_3 & \cdots & p_0 \end{Vmatrix}$$

where $1 > p_0 > 0$, and $p_0 + p_1 + \cdots p_N = 1$. Determine the limiting distribution for the process.

Problem 6. Let P be the transition probability matrix of a finite state regular Markov chain, and let Π be the matrix whose rows are the limiting distribution π. Define $Q = P - \Pi$.

a. prove $P^n = \Pi + Q^n$.

b. When

$$P = \begin{Vmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{Vmatrix},$$

please find an explicit expression for Q^n, and then for P^n.