MTH 543/653: Stochastic Modeling (Spring 2024)

Instructor: Songfeng (Andy) Zheng

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Room and Time: Cheek 173, 1:25pm – 2:15pm, MWF

Office and Hours: Cheek Hall 22M, 8:30am – 11:00am, Tuesday and Thursday; or by appointment. It is a better idea to formulate your question in email, and if it is easy to explain your question in email, I will reply to your email immediately when I see it. Alternatively, you can also make an appointment for a Zoom meeting, and the Meeting ID is **8514343306**.

Textbooks: An Introduction to Stochastic Modeling, 4-th Edition, by Taylor and Karlin. (Required)

Introduction to Probability Models, 10-th Edition, by S. Ross (Reference)

Course webpage:

http://people.missouristate.edu/songfengzheng/MTH543S24.htm will provide the download of various course materials, including the lecture notes, homework assignments, announcements, and data for exercises.

Objectives & Prerequisites: The course Math 543/653 is devoted to applications of probability and statistics from a modeling point of view. Topics include generating functions, branching processes, martingales, discrete time Markov chains, classification of states, estimation of transition probabilities, continuous time Markov Chains, Poisson processes, birth and death processes, renewal theory, queuing systems, Brownian motion and its properties, Gaussian process, and stationary processes. The prerequisite for this course is MTH 540, or equivalent; students are expected to be familiar with calculus (differentiation, integral, sum of infinite series).

Materials to be covered (tentative): Review of probability theory, moment generating functions, law of large numbers and central limit theorem; conditional probability and conditional expectation, discrete time Markov Chains, continuous time Markov Chains, branching process, exponential distribution, counting process, Poisson process, renewal process, Brownian motion, computer simulations: random variable generating and stochastic process implementation.

Grading Policy and Studying Suggestions:

Homework: 20% In-class Tests: 30% Final Exam: 50%

Grading policy: A (>90%) B (80 --- 89%) C (70 --- 80%) D (60 – 70%), F(<60%)

Final Exam date: 1:15pm --- 3:15pm, Monday, May 6, 2024.

Note: for students enrolled in MTH653, you are expected to finish one more question in each exam or finish several small projects!

It is important that you read the text book(s) regularly, understand the problems worked out in the text and practice by doing the problems. Doing the homework problems is absolutely essential to get a better grade and achieve a better understanding to the materials in this course. You are allowed to discuss the homework problems among yourselves or with me. However the final work handed in must be completely your own. Anyone who receives or gives an unauthorized aid on a homework or test is considered to be cheating.

No make-up test or exam will be given under ordinary conditions. The only acceptable excuse for missing a test is an extreme emergency. However, you must obtain a written explanation from a physician, etc. If you cannot take the test on the scheduled day, you must contact me before the test date.

Emailing format:

Email is an important means to communication in everyday life as well as in this course. Due to the large amount of emails sent to me every day, and due to different courses I am teaching, I suggest you clearly write a subject in the email, and in the subject, clearly tell which course you are from. For example, a good email subject would be like

Subject: MTH 543: Q about #7 in HW2

Thus, I can quickly locate your problem and will reply quickly. Emails which don't have a clear subject may be simple ignored!

Classes During Campus Closures

When the university is closed due to an emergency or inclement weather situation, classes will move to remote learning. What this means for our class is that we will meet via Zoom during our regularly scheduled class time OR you will complete and submit an alternative online assignment by the specified due date OR I will post alternative materials for you to read/review. I will use email to communicate any changes to scheduled tests, quizzes, or other assessments that may be impacted. If you are not able to participate in the remote learning activities as described (for

example, due to a power outage), you should contact me as soon as you can so alternative arrangements can be made.

Miscellaneous Notes:

Attendance policy: The University expects instructors to be reasonable in accommodating students whose absence from class resulted from: (1) participation in University-sanctioned activities and programs; (2) personal illness; or (3) family and/or other compelling circumstances. Instructors have the right to request documentation verifying the basis of any absences resulting from the above factors. Please see The University's attendance policy can be found in the *2010-2011 Undergraduate Catalog* at www.missouristate.edu/registrar/attendan.html.

Academic integrity: Missouri State University is a community of scholars committed to developing educated persons who accept the responsibility to practice personal and academic integrity. You are responsible for knowing and following the university's *Student Academic Integrity Policies and Procedures*, available at www.missouristate.edu/policy/academicintegritystudents.htm. You are also responsible for understanding and following any additional academic integrity policies specific to this class (as outlined by the instructor). Any student participating in any form of academic dishonesty will be subject to sanctions as described in this policy. If you are accused of violating this policy and are in the appeals process, you should continue participating in the class.

Nondiscrimination: Missouri State University is an equal opportunity/affirmative action institution, and maintains a grievance procedure available to any person who believes he or she has been discriminated against. At all times, it is your right to address inquiries or concerns about possible discrimination to the Office for Equity and Diversity, Park Central Office Building, 117 Park Central Square, Suite 111, (417) 836-4252. Other types of concerns (i.e., concerns of an academic nature) should be discussed directly with your instructor and can also be brought to the attention of your instructor's Department Head. Please visit the OED website at www.missouristate.edu/equity/.

Disability Accommodation: If you are a student with a disability and anticipate barriers related to this course, it is important to request accommodations and establish an accommodation plan with the University. Please contact the Disability Resource Center (DRC) (https://www.missouristate.edu/disability/), Meyer Library, Suite 111, 417-836-4192, to initiate the process to establish your accommodation plan. The DRC will work with you to establish your accommodation plan, or it may refer you to other appropriate resources based on the nature of your disability. In order to prepare an accommodation plan, the University usually requires that students provide documentation relating to their disability. Please be prepared to provide such documentation if requested. Once a University accommodation plan is

established, you may notify the class instructor of approved accommodations. If you wish to utilize your accommodation plan, it is suggested that you do so in a timely manner, preferably within the first two weeks of class. Early notification to the instructor allows for full benefit of the accommodations identified in the plan. Instructors will not receive the accommodation plan until you provide that plan, and are not required to apply accommodations retroactively.

Cell phone policy: As a member of the learning community, each student has a responsibility to other students who are members of the community. When cell phones or pagers ring and students respond in class or leave class to respond, it disrupts the class. Therefore, the Office of the Provost prohibits the use by students of cell phones, pagers, PDAs, or similar communication devices during scheduled classes. All such devices must be turned off or put in a silent (vibrate) mode and ordinarily should not be taken out during class. Given the fact that these same communication devices are an integral part of the University's emergency notification system, an exception to this policy would occur when numerous devices activate simultaneously. When this occurs, students may consult their devices to determine if a university emergency exists. If that is not the case, the devices should be immediately returned to silent mode and put away. Other exceptions to this policy may be granted at the discretion of the instructor.

Emergency Response policy: Students who require assistance during an emergency evacuation must discuss their needs with their professors and Disability Services. If you have emergency medical information to share with me, or if you need special arrangements in case the building must be evacuated, please make an appointment with me as soon as possible. For additional information students should contact the Disability Resource Center, 836-4192 (PSU 405), or Larry Combs, Interim Assistant Director of Public Safety and Transportation at 836-6576. For further information on Missouri State University's Emergency Response Plan, please refer to the following web site: http://www.missouristate.edu/safetran/erp.htm

Dropping a Class: It is your responsibility to understand the University's procedure for dropping a class. If you stop attending this class but do not follow proper procedure for dropping the class, you will receive a failing grade and will also be financially obligated to pay for the class. For information about dropping a class or withdrawing from the university, contact the Office of the Registrar at 836-5520.

Changes to this syllabus: The instructor reserves the right to make changes to this syllabus, and the changes will be announced in class and on the course webpage.

Tentative Lecture Schedule in Spring 2024 (MTH 543/653)

Lecture 1: Experiment, sample space, event, operation of events, Axioms of probability, conditional probability, independence. Example.

Lecture 2: random variables, discrete random variable and commonly used discrete distributions.

Lecture 3: Continuous random variable and commonly used continuous distributions, expectation, variance.

Lecture 4: Joint distribution of random variables, independent random variables, covariance and properties.

Lecture 5: Cauchy-Schwarz inequality. moment generating function and properties.

Lecture 6: Markov Inequality, Chebyshev Inequality, Laws of large numbers, Central Limit theorem.

Lecture 7: Conditional distribution and conditional expectation, examples.

Lecture 8: Conditional expectation, wald's formula, using conditional expectation to find expectations.

Lecture 9: Conditional Variance, VEV formula, properties of conditional expectation, Examples.

Lecture 10: Stochastic process, martingale, property, examples and counter-examples.

Lecture 11: Property of Martingale, Markov Inequality, maximal inequality for martingale and proof.

Lecture 12: Counting Process and properties, two definitions of Poisson process.

Lecture 13: understanding of the Poisson process, the equivalence between the two definitions.

Lecture 14: Waiting time, Sojourn time. The distribution of the waiting time and its proof.

Lecture 15: Sojourn time and their distributions, distribution of number of events during a period of time; order statistic, joint distribution of order statistic for uniform random variables.

Lecture 16: Conditional joint distribution of the waiting time, connection with the uniform random variable, proof.

Lecture 17: Some applications of Poisson process.

Lecture 18: Application of Poisson process. Compound Poisson process, non-homogeneous Poisson process.

Lecture 19: Renewal Process and related concepts, Poisson process as a special case of Renewal process.

Lecture 20: Markov Chain, transition probability matrix, Chapman-Kolmogorov Equation, state distribution after n step.

Lecture 21: determine state probability given the initial state, examples of Markov chain.

Lecture 22: Random Walk models; Queuing Markov chain.

Lecture 23: First step analysis, example.

Lecture 24: One dimensional Random walk and gambler's ruin

Lecture 25: solution to the Gambler's ruin problem.

Lecture 26: Branching process. Regular transition probability matrix, limiting distribution.

Lecture 27: Classification of states; communicative and equivalent classes.

Lecture 28: Recurrent and Transient states, probability of first return time, rules to determine recurrent and transient.

Lecture 29: recurrent transient in 1D and 2D random walk.

Lecture 30: Continuous time Markov process, Birth process.

Lecture 31: Probability of the population size at any time, the corresponding differential equations and proof.

Lecture 32: Example of Birth Process. Pure Death process and example.

Lecture 33: Birth and Death process, definition and explanation. Infinitesimal generator. Kolmogorov backward equations.

Lecture 34: Deriving the Kolmogorov's backward and forward differential equations.

Lecture 35: Matrix form of Kolmogorov's backward and forward differential equations, Example. Brief history of Brownian motion.

Lecture 36: limiting case of 1D symmetric random walk, definition of Brownian motion, mean and variance of Brownian motion, covariance at two time points.

Lecture 37: multivariate normal distribution, Gaussian process, Brownian motion as Gaussian process.

Lecture 38: Determining Brownian motion by Gaussian process, Examples. Symmetric property of Brownian motion, Brownian motion and heat equation.

Lecture 39: Brownian motion as a continuous time martingale. Examples.

Lecture 40: first hitting time of Brownian motion.

Lecture 41: Maximum value of Brownian motion. zeros of Brownian motion, and arcsin law.

Lecture 42: Proof of Arcsin law, non-differentiability of path of Brownian motion.

Lecture 43: Brownian motion with drift.