Random Experiment

- In random experiments, the result is unpredictable, unknown prior to its conduct, and can be one of several choices.

- Examples:
  - The Experiment of tossing a coin (head, tail)
  - The Experiment of rolling a die (1,2,3,4,5,6)
  - (More Examples) _________________________
Sample Space

- The enumeration of all possible outcomes of an experiment is called the **sample space**, denoted $S$.
  
  E.g.: $S=\{\text{head, tail}\}$

- Collection of some outcomes is called an **event** and usually denoted with capital letters (e.g., A, B, C).

- Individual events are called **simple events**.
  
  E.g.: $\{\text{head}\}$, $\{\text{tail}\}$
Rolling a fair die

- What are the possible outcomes? ___________________
- Sample space: $S = \{1, 2, 3, 4, 5, 6\}$
- Is it more likely that I get “1” than “2”? Yes   No
- Can we assume that each outcome in this experiment is equally likely to occur? Yes   No
- How likely? _________
- $1/6$
To describe the likelihood that some outcome or event A occurs, we assign to this event a numeric value, called *probability* and denoted $P(A)$.

Some major properties of $P(A)$:

- $0 \leq P(A) \leq 1$
- An impossible event has a probability of 0, $P(A) = 0$
- A certain event has a probability of 1, $P(A) = 1$
- The probability that at least one event from the sample space $S$ occurs is 1, $P(S) = 1$
Outcomes that equally likely to occur

- **Sample space** $S$ that contains $N$ possible unique outcomes
- Each outcome in an experiment is equally likely to occur, the **probability of each outcome** is given by:
  \[ P(\text{outcome}) = \frac{1}{N} \]
- In this case, the **probability of an event** is given by:
  \[ P(\text{event}) = \frac{\text{(# outcomes in event)}}{N} \]
Rolling a fair die

- What is the probability that you roll an even number?
- Define an event $A =$ ”roll an even number” and list all possible outcomes for event $A = \{2, 4, 6\}$.
- Compute
  \[
  P(A) = \frac{\text{# outcomes in } A}{N} = \frac{3}{6} = \frac{1}{2}
  \]
- $1/2$
Rolling a fair die

- Define an event $C =$” roll an odd number” and list all possible outcomes for event $C = \{1, 3, 5\}$
- Compute $P(C)$
  - $1/2$
- $A =$”roll an even number”
- Do $A$ and $C$ have any common outcomes?
  - No
Mutually Exclusive (Disjoint) Events

Two events A, C are **mutually exclusive** (or **disjoint**) if they do not contain any common outcomes.

- Are events A =”roll an even number” and C =”roll an odd number” mutually exclusive?
  - Yes
- List all outcomes that are contained in the sample space S, but not in event A
  - \{1,3,5\}
- Is this event is the same as event C?
  - Yes
- C is the **complement** of A
The complement of an event $A$ consists of all outcomes in the sample space $S$ that are not in the event $A$, and denoted $A'$. 

- $P(C)$ can be also computed using the Complement Rule: $P(C) = 1 - P(A)$

- Venn Diagram:
Example

<table>
<thead>
<tr>
<th>Gender</th>
<th>Freshmen</th>
<th>Sophomore</th>
<th>Junior</th>
<th>Senior</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>15</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>Female</td>
<td>49</td>
<td>19</td>
<td>8</td>
<td>5</td>
<td>81</td>
</tr>
<tr>
<td>Total</td>
<td>64</td>
<td>29</td>
<td>9</td>
<td>6</td>
<td>108</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected student is:

- Freshmen \( P(F) = \) _______________________
- Not Freshmen \( P(F') = \) _______________________


What is the probability that a randomly selected student is:

- **Freshmen**  \( P(F) = \frac{64}{108} = 0.5926 \)

- **Not Freshmen**  \( P(F') = 1 - 0.5926 = 0.4074 \)
Example

- \( J = \) “Junior” ,  \( F = \) “Freshmen”
- Are events \( J \) and \( F \) mutually exclusive?
  - Yes
- Are they complementary?
  - No
Imagine a game...

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- What is the probability that you win 20$?
- What is the probability that you win at least 10$?
- What is the probability that you win 10$?
Imagine a game...

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- Denote $A =$”roll an even number” $= \{2, 4, 6\}$
  - $P(A) = 0.5$

- $B =$”roll 4 or greater” $= \{4, 5, 6\}$
  - $P(B) = 0.5$
Imagine a game…

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- What is the probability that you win 20$?
  - A =”roll an even number”, B =”roll 4 or greater
  - D = “you win 20$” - events A and B occur simultaneously.
  - D = {_____________}
  - P(D) = ____________
Imagine a game...

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- What is the probability that you win 20$?
  - A = ”roll an even number”, B = ”roll 4 or greater
  - D = “you win 20$” - events A and B occur simultaneously.
  - D = {4, 6}
  - P(D) = 2/6 = 1/3
  - D is the intersection of events A and B.
Intersection

- The **intersection** of two events A and B denoted \( A \cap B \) consists of outcomes that are in both events.

- The probability of intersection of two events A and B called the **joint probability**, \( P(A \cap B) \).
Imagine a game…

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- What is the probability that you at least 10$?
  - A =”roll an even number”,  B =”roll 4 or greater
  - D = “you win at least 10$” - events A or B occur simultaneously.
  - D = {____________________________}  
  - P(D) = _________________
  - D is the **union** of events A and B.
Imagine a game...

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.
- What is the probability that you at least 10$?
  - A = ”roll an even number”, B = ”roll 4 or greater
  - D = “you win at least 10$” - events A or B occur simultaneously.
  - D = {2, 4, 5, 6}
  - P(D) = 4/6 = 2/3
  - D is the union of events A and B.
The **union** of two events A and B denoted consists of outcomes that are in either event (or in both events).

**The Addition Rule:**

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
Imagine a game...

- You will win $10 if you roll an even number and $10 if you roll 4 or a greater number.

- What is the probability that you exactly $10?
  - A ="roll an even number", B ="roll 4 or greater"
  - D = “you win exactly least $10”
  - D = {______________}
  - P(D) = ______________
  - Define D using the notation of A, B, and logic operands: or, and, complement.
Imagine a game...

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- What is the probability that you exactly 10$?
  - A =”roll an even number”, B =”roll 4 or greater”
  - D = “you win exactly least 10$”
  - D = \{2,5\}
  - P(D) = 2/6 = 1/3
  - D = (A or B) and (A and B)’
**Example**

**M** = “Male”,  **F** = “Freshmen”

What is the probability that a randomly chosen student is male freshmen?

P(M and F) = P(MF) = ________________

P(M and F) = 15/108 = 0.1389
Conditional Probability

- **Conditional probability** is the probability of some event $A$, given the occurrence of some other event $B$.

- **Conditional probability**, $P(A|B)$, is read "the (conditional) probability of $A$, given $B" or "the probability of $A$ under the condition $B". 
Conditional Probability

- When in a random experiment the event $B$ is known to have occurred:
  - possible outcomes of the experiment are reduced to $B$,
  - hence $P(A)$ is changed into $P(A|B)$. 

Marginal Probability

- **Marginal probability** is the unconditional probability $P(A)$ of the event $A$.

- $P(A)$ is the probability of $A$, regardless of whether event $B$ did or did not occur.
The Conditional Probability Formula

\[ P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)} \]
Imagine a game…

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.

- What is the probability that you exactly 10$?
  - A =”roll an even number”, B =”roll 4 or greater”
  - P(A and B) = _________________
  - P(B) = _____________________

\[
P(A|B) = \frac{P(A \cap B)}{P(B)} = \text{____________________} =
\]
Imagine a game…

- You will win 10$ if you roll an even number and 10$ if you roll 4 or a greater number.
- What is the probability that you exactly 10$?
  - A =”roll an even number”, B =”roll 4 or greater”
  - P(A and B) = 1/3
  - P(B) = 1/2
  - P(A|B) = 2/3
  - P(A) =1/2 < P(A|B)