MTH 507/607:
Introduction to Partial Differential Equations

Instructor: William O. Bray
Office: Cheek 8M
Office Hours: MWF 1-2 PM & by appointment
EMAIL
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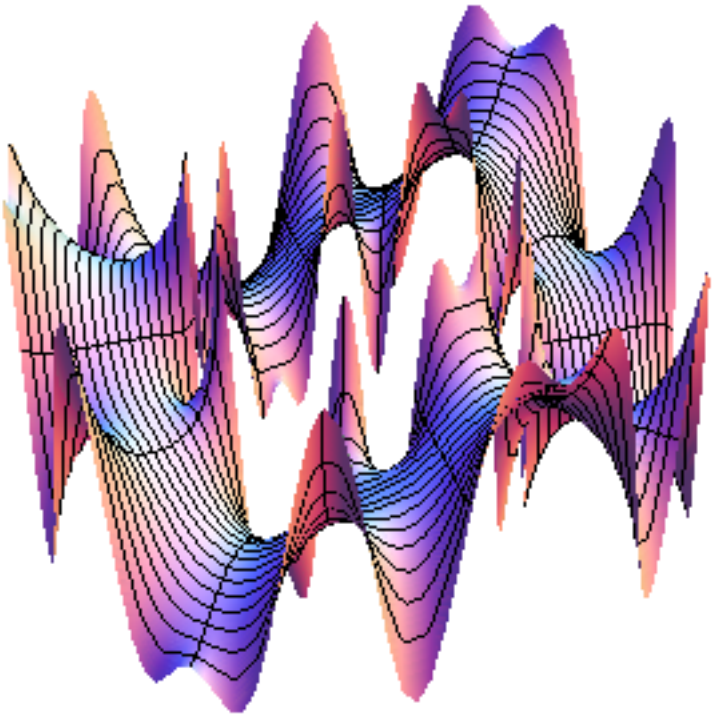
Dirichlet Problem in an Annulus



If one looks at the different problems of the integral calculus which arise naturally when one wishes to go deep into the different parts of physics, it is impossible not to be struck by the analogies existing. Whether it be electrostatics or electrodynamics, the propagation of heat, optics, elasticity, or hydrodynamics, we are led always to differential equations of the same family. (Henri Poincare, circa 1890)

Overview

The study of PDE's arose in 18th century in attempts to understand and model physical phenomena such as heat flow and vibrations of elastic strings and membranes. Indeed, partial differential equations find application in virtually all physical phenomena: quantum mechanics, cellular processes, thermodynamics, mechanics, and relativity. From a mathematical perspective, the study of partial differential equations has had a cornerstone impact in virtually all areas of mathematics: analysis, algebra, geometry, probability and stochastic processes.

The focus of this course is the study of the classical trinity of linear partial differential equations : the heat equation, the wave equations, and Laplace's equation. We will come to understand some of the physics underlying these equations and study them from qualitative and quantitative methods. Both of the latter will involve ideas from geometry and analysis and efforts will be made to bring out the interconnections between these as well as their connections with the underlying physics. For linear equations, a principle of superposition is core and motivates the study of an area of mathematics known as harmonic analysis, aka Fourier analysis. A key learning tool for us will be to use the interplay between physical principles underlying the aforementioned equations and the ideas from geometry and harmonic analysis underlying their solution.

Specific course goals include:

- Understand first order linear and nonlinear PDE's, some of their underlying physics, and solution methods.
- Understand second order linear PDE's, in particular the classical trinity of equations: diffusion or heat equation, wave equation, potential equation.
- Understand the role of boundary and initial conditions in PDE problems and their connection with physics.
- Understand the fundamental questions in our study: Existence, Uniqueness, Stability
  and their pragmatic versions:
  Construction of Solution Formula, Regularity of Solutions, Approximation of solutions.
- Understand the concept of a distribution or generalized function and it's role in PDE problems.

- Be able to make use of qualitative Methods: vector calculus, energy methods, maximum principles.
- Be able to make use of quantitative Methods: symmetries of equations, separation of variables, Fourier series, eigenfunction expansions, distributions and Green's functions.

MTH 607 Students enrolled in the graduate version of this course, MTH 607, will demonstrate a deeper understanding of the above goals through problem sets and exam questions aimed at bringing in graduate level mathematics.

Text William O. Bray, *A Journey into Partial Differential Equations*, Jones & Bartlett Learning (2011)

Software Throughout the course we will make use of *Mathematica* (blue = hyperlink) for the purposes of graphic illustration and computation. Some homework assignments/projects will be given in the form of Mathematica notebooks. Mathematica is available on all campus clusters and student software licenses are available. (NOTE: You will have to create an account at this link with Wolfram Research.) A great screencast introduction to using Mathematica is found at: Hands-on Start to Mathematica. Other useful screencasts are at: Mathematica Screencasts. Mathematica notebooks will be used at various times in class and will be posted for your use and reference.

Assessment

Homework, 50% Assignments will be given regularly taken from the text and in the form of handouts. Assigned homework is to be done individually and written up in neat form using complete sentences. Any figures or drawings should be clearly labeled.

Projects, 15% I plan on having one or two collaborative projects over the course of the semester. These can be done in groups of 2-3 with one write up for each group.

In Class Exam 15% This will be given before the break in March.

Final, 20% A take-home final will be given a week before classes end and will be due at the end of finals week. Of course, this is to be done individually.

NOTE I may use $\pm$ grades in borderline cases when awarding your course grade.

Miscellaneous

Attendance/Participation I expect you to attend every class and become engaged in discussion periods and problem solving. If you need to miss class due to illness, family emergency, or other reasonable reason, please let me know.

Nondiscrimination Missouri State University is an equal opportunity/affirmative action institution, and maintains a grievance procedure available to any person who believes he or she has been discriminated against. At all times, it is your right to address inquiries or concerns about possible discrimination to the Office for Institutional Equity and Compliance, Park Central Office Building, 117 Park Central Square, Suite 111, (417) 836-4252. Other types of concerns (i.e., concerns of an academic nature) should be discussed directly with your instructor and can also be brought to the attention of your instructor's Department Head. Please visit the OED website at www.missouristate.edu/equity/.

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Academic Dishonesty Missouri State University is a community of scholars committed to developing educated persons who accept the responsibility to practice personal and academic integrity. You are responsible for knowing and following the university's student honor code, Student Academic Integrity Policies and Procedures and also available at the Reserves Desk in Meyer Library. Any student participating in any form of academic dishonesty will be subject to sanctions as described in this policy.

Cell Phones Let's keep it simple: they are disruptive and interfere with the learning process. Leave them off/on vibrate and put away.

Emergency Response Please familierize yourself with the following emergency response sites:
http://www.missouristate.edu/safetran/51597.htm
http://www.missouristate.edu/safetran/erp.htm