MTH 701 Real Analysis

William O. Bray
Office: Cheek 8M
Office Hours: 1:30-2:30 MW & by appointment

Fall 2014

Historical Perspectives

In the late 18th to early 19th centuries, a fundamental question concerned the representability of a function in a series of sines and cosines of various frequencies. This question arose from the study of heat flow and wave phenomena. As a specific example, for a function $f(x)$ defined on $[0, \pi]$, determine the coefficients $\{a_n\}$ such that

$$f(x) = \sum_{n=1}^{\infty} a_n \sin nx.$$

In this time period, the notion of a function was vague to say the least and convergence of infinite series was still in its infancy. None the less, Newton and Liebniz had introduced the concept of derivative and by manipulating antiderivative formulas, several examples of functions for which the above series representation held were known.

This question was given more solid foundation by Bernard Riemann in 1850. Indeed this question was his motivation for developing the definite integral that bears his name. In this context, the solution to the above question takes the form

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx,$$

known as the Fourier sine coefficients of f. In the beginning of the 20th century Henri Lebesgue realized that deeper analysis of this question was connected with the problem of interchange of sequential limit operations and definite integrals, i.e., under what (natural) conditions on the sequence $\{f_n(x)\}$ of functions defined on an interval $[a, b]$ is it true that:

$$\lim_{n \to \infty} \int_a^b f_n(x) \, dx = \int_a^b \lim_{n \to \infty} f_n(x) \, dx?$$

His analysis lead to the conception of Lebesgue measure and integral, the principle topic of this course. Aside from Riemann and Lebesgue, several other mathematicians should be mentioned: J. Fourier, J.P. Dirichelt, E. Borel, M. Riesz, J. Radon, M. Frechet, G. Fubini, C. Caratheodory, F. Hausdorff, and A. Besicovitch. Many other historical perspectives can be gleaned from the last two references below.

Course Outline

Topics The following is a brief list of topics contained in the first eight chapters of the principal text listed below.

- The Riemann integral and its weaknesses.
- The Riemann-Stieljes integral.
- Lebesgue measure and measurable functions: theorems of Lusin and Egoroff.
- The Lebesgue integral and convergence theorems.
- Repeated integration & change of variables.
- L^p-spaces and inequalities.

Expectations Written homework 50%, Midterm 25%, Final 25%. For all written work I encourage you to download the program Lyx (www.lyx.org); it is relatively easy to use and generates nice pdf output for your solutions. I will demo the program during the first week of classes and will be using it to provide homework sets (both as .lyx and .pdf files; the latter in case you submit your homework using other mechanisms).

References

Our primary text will be the first item below. I will also be handing out course notes occasionally.

- R.L. Wheeden & A. Zygmund, *Measure and Integral,* Marcel Dekker (1977). **Available on Amazon!!**
- M.E. Taylor, *Measure Theory and Integration,* Amer. Math. Soc. Graduate Studies Volume 76 (2006).
- H.L. Royden, P.M. Fitzpatrick, *Real Analysis,* 4th edition, Prentice Hall (2010).
- T. Hawkins, *Lebesgue's Theory of Integration: It's Origins and Development,* American Math. Soc. (2002).
- William Durham, *The Calculus Gallery: Masterpieces from Newton to Lebesgue,* Princeton Univ. Press (2008).

The latter two references are historical in nature and I may draw from for course filler.

Miscellaneous

Attendance/Participation I expect you to attend every class and become engaged in discussion periods and problem solving. If you need to miss class due to illness, family emergency, or other reasonable reason, please let me know.

Nondiscrimination Missouri State University is an equal opportunity/affirmative action institution, and maintains a grievance procedure available to any person who believes he or she has been discriminated against. At all times, it is your right to address inquiries or concerns about possible discrimination to the Office for Institutional Equity and Compliance, Park Central Office Building, 117 Park Central Square, Suite 111, (417) 836-4252. Other types of concerns (i.e., concerns of an academic nature) should be discussed directly with your instructor and can also be brought to the attention of your instructor's Department Head. Please visit the OED website at www.missouristate.edu/equity/.

Disability Accommodation To request academic accommodations for a disability, contact the Director of the Disability Resource Center, Plaster Student Union, Suite 405, (417) 836-4192 or (417) 836-6792 (TTY), www.missouristate.edu/disability. Students are required to provide documentation of disability to the Disability Resource Center prior to receiving accommodations. The Disability Resource Center refers some types of accommodation requests to the Learning Diagnostic Clinic, which also provides diagnostic testing for learning and psychological disabilities. For information about testing, contact the Director of the Learning Diagnostic Clinic, (417) 836-4787, http://psychology.missouristate.edu/ldc.

Academic Dishonesty Missouri State University is a community of scholars committed to developing educated persons who accept the responsibility to practice personal and academic integrity. You are responsible for knowing and following the university's student honor code, Student Academic Integrity Policies and Procedures and also available at the Reserves Desk in Meyer Library. Any student participating in any form of academic dishonesty will be subject to sanctions as described in this policy.

Cell Phones Let's keep it simple: they are disruptive and interfere with the learning process. Leave them off/on vibrate and put away.

Emergency Response See http://www.missouristate.edu/safetran/51597.htm