Problem 1: (#3 on page 369 of DeGroot) Suppose that random variables $X_1, \ldots, X_n$ are independent and $X_i$ has an exponential distribution with parameter $\beta$ unknown. Determine the MLE of the median of the distribution.

Problem 2: (#5 on page 369 of DeGroot) Suppose that random variables $X_1, \ldots, X_n$ are independently from the uniform distribution on $[a, b]$. Find the MLE of the mean of the distribution.

Problem 3: (#8 on page 369 of DeGroot) Suppose that random variables $X_1, \ldots, X_n$ are independent and $X_i$ has a gamma distribution with the p.d.f. as $\alpha$ unknown

$$f(x|x_0, \alpha) = \begin{cases} \frac{1}{\Gamma(\alpha)} x^{\alpha - 1} e^{-x}, & \text{for } x \geq x_0 \\ 0, & \text{otherwise} \end{cases}$$

Find the MLE of $\Gamma'(\alpha)/\Gamma(\alpha)$.

Problem 4: (#9 on page 369 of DeGroot) Suppose that random variables $X_1, \ldots, X_n$ are independent and $X_i$ has a gamma distribution with the p.d.f. as $\alpha, \beta$ unknown

$$f(x|x_0, \alpha) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\beta x}, & \text{for } x \geq x_0 \\ 0, & \text{otherwise} \end{cases}$$

Find the MLE of $\alpha/\beta$. 