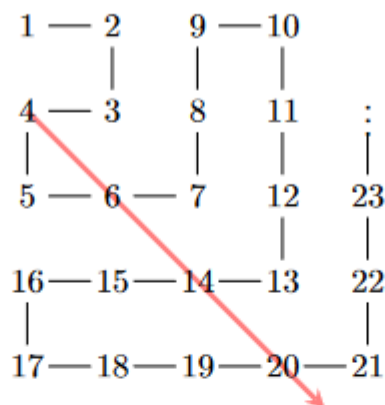


S4. What is the 19th term in the sequence 4, 6, 14, 20, ... determined by a diagonal line (in red) in the following diagram:



We received 6 submissions, all correct. We present the solution by Missouri Problem Solving Group.

The paths below the diagonal (e.g., from 4 to 6 and from 14 to 20) increase by 4 steps with each iteration, so the path lengths would be 2, 6, 10, 14, ... The paths above the diagonal (e.g. from 6 to 14 and from 20 to 32) also increase by 4 steps with each iteration, so those path lengths would be 8, 12, 16, 20, ... Denote the entries on the diagonal by a_1, a_2, a_3, \dots . The lengths of the paths between entries with odd indices are $2 + 8, 6 + 12, 10 + 16, \dots$ or $10, 18, 26, \dots$. The k th term in this sequence is $8k + 2$ and

$$a_{2n+1} = a_1 + \sum_{k=1}^n (8k + 2) = 2 + 8 \cdot \frac{n(n+1)}{2} + 2n = 4n^2 + 6n + 4.$$

Therefore $a_{19} = 4 \cdot 9^2 + 6 \cdot 9 + 4 = 382$.

A similar argument shows that $a_{2n} = 4n^2 + 2n$.